Appendix

Temperature of the Stratopause

We have remarked on a number of occasions that the high temperature near 50 km is caused by absorption of solar radiation by ozone. Let us examine this statement a little further in terms of Fig. 3–9, which refers to the transparent outer fringe of the atmosphere.

We drop the assumption that solar radiation passes directly to the Earth's surface without absorption. Instead, we allow the thin layer of atmosphere to absorb visible solar radiation with an opacity δ. For infrared planetary radiation (see Fig. 3–4), the opacity of the layer is ε. The solar flux can be equated to σT_e^4, so that $\delta\sigma T_e^4$ is the amount of solar energy absorbed in the layer.

The balance of energy is now altered. We must write

Energy absorbed from solar radiation + Energy absorbed from infrared planetary radiation = Energy lost by infrared radiation

or

$$\delta\sigma T_e^4 + \varepsilon\sigma T_e^4 = 2\varepsilon\sigma T_s^4$$

therefore

$$\left(\frac{T_s}{T_e}\right)^4 = \frac{\delta + \varepsilon}{2\varepsilon}$$

Our previous discussion corresponded to $\delta = 0$ (no absorption of solar radiation in the atmosphere) so the skin temperature was given by $(T_s/T_e)^4 = \frac{1}{2}$. The optical properties of the atmosphere for solar and planetary radiation, however, are quite different. In particular, the opacity for solar radiation can be very great indeed at some wavelengths. Thus, δ can be greater than ε at high altitudes, leading to higher temperatures at the altitudes where solar radiation is absorbed.

In this way we can understand the existence of warm layers in the atmosphere. Figure 2–4 tells us that the warm layer caused by ozone absorption should occur close to 50 km above the ground. Detailed calculations of the temperature profile give good agreement between theory and observation.

The Adiabatic Lapse Rate

A change in which no heat transfer takes place is called *adiabatic*. We wish to discover the rate of variation of temperature with height consistent with adiabatic motions; we have already demonstrated that the temperature will decrease with height.

In Fig. A–1 we wish to move a parcel of air adiabatically from height z_1 (pressure p_1, temperature T_1) to height z_2 (pressure p_2, temperature T_2), without exchanging any heat with the surrounding atmosphere. We wish to find the value of the adiabatic lapse rate,

$$\Gamma = -\left(\frac{T_2 - T_1}{z_2 - z_1}\right)$$

Consider a parcel consisting of a single gram of air. When it undergoes any change of height, temperature, or pressure, the first law of thermodynamics states that the total energy in all identifiable forms (erg gm^{-1}) must not change. Let us consider what these changes are:

1. Q = heat brought into the parcel. For an adiabatic change, $Q = 0$.
2. E = increase of internal energy of the molecules. It is a property of a perfect gas that internal energy depends on the temperature only. Thus, for a change at constant temperature, $E = 0$.
3. W = work done on the parcel. Since the gas is compressible, the boundaries of the parcel will move inward if pressure is applied. The parcel will contract and work will be done on it. For an incompressible substance, however, $W = 0$.
4. P = increase of potential energy in the gravitational field. This is the work that could be realized in falling from z_2 to z_1. The force on 1 gm (dyne gm^{-1}) is $g \times 1$ (cm sec^{-2}). Work is force times distance. Thus, the potential energy of a gram at z_1 is less than that at z_2 by the amount $g \times (z_2 - z_1)$ ($\text{cm}^2\ \text{sec}^{-2}$ equal to erg gm^{-1}).

The first law of thermodynamics, in symbolic form, states that for any change

$$P + E = Q + W$$

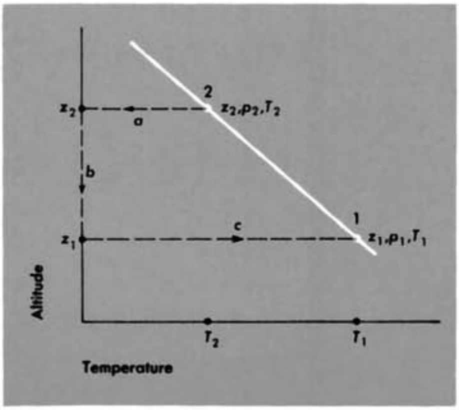

FIGURE A-1. *Evaluation of the adiabatic lapse rate.*

If we pass 1 gm of air directly from 2 to 1, we must have $Q = 0$, since the change is adiabatic. To evaluate the terms P, E, and W, we will go from 2 to 1 by changes a, b, and c. Changes a and c take place at constant height, or constant pressure (the two are related through the barometric law). On the other hand, b is a change of height at the absolute zero of temperature.

Consider changes a and c. The amount of heat required to change the temperature of 1 gm of air by 1°K is called the *specific heat* (erg gm^{-1} deg^{-1}). If the change takes place at constant pressure, this specific heat is known as c_p, the specific heat at constant pressure. It is an important property of an ideal gas that c_p is a constant, and does not depend upon temperature or pressure. This is, in effect, an evaluation of the two terms $(W - E)$. The term P we know to be zero if the height does not change. Hence, for a,

$$Q_a = -\, c_p T_2 \text{ (heat is removed)}$$

and for c

$$Q_c = +c_p T_1 \text{ (heat is added)}$$

For step b, $E = 0$ because the change is at constant temperature. Now, as we are aware from common experience, at very low temperatures a gas will assume the high density of a solid. In fact, if the ideal gas law were obeyed at all temperatures, the density would become infinitely large at $T = 0$. The important point is that when the parcel is small and very dense it is effectively incompressible, like a solid or a liquid. Hence for change b, $W = 0$, and

$$Q_b = P = -\, g(z_2 - z_1)$$

If the three changes together are to be adiabatic, we must have

$$Q_a + Q_b + Q_c = 0$$

Therefore

$$c_p\,(T_1 - T_2) - g(z_2 - z_1) = 0$$

and

$$\Gamma = -\left(\frac{T_2 - T_1}{z_2 - z_1}\right) = g/c_p$$

Suggestions for further reading

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